Unimodular bimode gravity, grand unification and the scalar-graviton dark matter

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Abstract

In prior article of the author, the unimodular bimode gravity/*systo-gravity*, with the scalar-graviton/*systolon* dark matter, was worked out. To compile with the anomalous rotation curves of galaxies the scale of the local scale violation in the theory was shown to be about 10^{15} GeV. In this letter, to naturally incorporate such a scale the *hyper unification* framework, merging systo-gravity with grand unification through matter, is constructed. The systolon, as a free propagating compression mode in metric, emerges only below the unification scale, possessing at the same time a modified high-energy behaviour to be manifested at the high temperatures.

1 Introduction

Conceivably, General Relativity (GR) is not the fundamental theory of gravity being rather a (principle) part of the effective field theory of metric, which may a priori comprise other propagating gravity modes in addition to the (massless transverse-tensor) graviton. In this vein, in looking for a gravitational dark matter (DM) interior to metric, the unimodular bimode gravity (UBG) was proposed [1]. UBG explicitly violates the general gauge invariance/relativity to the residual unimodular one remaining still general covariant. Due to violation of the local scale invariance there emerges in the metric a (light) scalar graviton/systolon treated as DM. It proves that in order to compile with the galaxy dark halos the scale of violation of the local scale invariance should lie in the grand unified theory (GUT) range, about 10^{15} GeV. So the question arises whether this could be more then just a coincidence? This letter is an extension to [1]. Here we try to answer the posed question in affirmative by embedding UBG into the GUT framework. To make the exposition self-contained as far as possible, we first recapitulate the essence of UBG, necessary for the following,[1] merging then UBG and GUT, with the natural emergence of the required scale.

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[1]For more detail, see [1].

2 UBG/systo-gravity

Comprising the systolon and graviton, UBG will otherwise be referred to as the *systo-gravity*. The proper Lagrangian looks generically like

$$L_{sg} = L_g + L_s, \tag{1}$$

with contributions of graviton, L_g, and systolon, L_s, respectively, preserving and violating the general invariance/relativity. As a paradigm, choose for L_g the GR Lagrangian

$$L_g = -\frac{1}{2}\kappa_g^2 R, \tag{2}$$

where R is the Ricci scalar for metric $g_{\mu\nu}$. Here one puts $\kappa_g = 1/\sqrt{8\pi G} = 1.4 \times 10^{18}$ GeV, with G being the Newton constant. Some general invariant modification of L_g, say, through a function $f(R)$ is a priori admitted.

Let now the general invariance/relativity be violated to the residual unimodular one, $G \to U$, by means of the explicit dependence of L_s on the scalar density $g = \det g_{\mu\nu}$. To maintain the general covariance g should enter through g/g_*, where g_* is a non-dynamical scalar density of the same weight as the dynamical g. More particularly, without loss of generality represent systolon by the unimodular invariant and general covariant (dimensionless) scalar field

$$\varsigma = \ln \sqrt{-g}/\sqrt{-g_*}. \tag{3}$$

A priori, L_s is an arbitrary function of ς. To terminate the arbitrariness let us enhance U by a dynamical global symmetry defined in any fixed coordinates x^μ (with no coordinate change) through the field substitutions:

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\begin{aligned}
g_{\mu\nu}(x) &\rightarrow \bar{g}_{\mu\nu}(x) = e^{-2\lambda_0} g_{\mu\nu}(e^{-\lambda_0}x), \\
g(x) &\rightarrow \bar{g}(x) = e^{-8\lambda_0} g(e^{-\lambda_0}x),
\end{aligned}
\tag{4}
$$

with an arbitrary parameter λ_0, whereas

$$g_*(x) \to \bar{g}_*(x) = g_*(e^{-\lambda_0}x). \tag{5}$$

The symmetry is aimed at distinguishing the non-dynamical and dynamical fields. For the latter ones these transformations (followed by the coordinate substitutions $x^\mu \to \bar{x}^\mu = e^{\lambda_0}x^\mu$) coincide with the global scale transformations as a part of the general coordinate transformations. For this reason the general invariant part of L_{sg}, L_g, is also global symmetric. Eqs. (4) and (5) may be referred to as the compression transformations. In these terms, the non-dynamical measure $\sqrt{-g_*}$ is "incompressible" in contrast to the dynamical one $\sqrt{-g}$. It follows thereby that ς transforms inhomogeneously under compressions:

$$\varsigma(x) \to \bar{\varsigma}(x) = \varsigma(e^{-\lambda_0}x) - 4\lambda_0. \tag{6}$$

In the so-called "transverse" coordinates/gauge, where $g_* = \bar{g}_* = -1$, compressions are equivalent to the global scale transformations/dilatations.[2]

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[2] Under the (unbroken) compression symmetry, the systolon may be considered as a general covariant counterpart of dilaton, the latter being the Goldstone boson corresponding to the (non-linearly realized broken) dilatation symmetry. Clearly, their underlying motivations and meanings are different.

Such a (approximate) global symmetry may serve as a *raison d'être* for suppression of the derivativeless couplings of systolon. In terms of the dimensionfull scalar field $\sigma \equiv \kappa_s \varsigma$, the admitted Lagrangian looks like

$$L_s = \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + V_s(\sigma), \tag{7}$$

where $\kappa_s \leq \kappa_g$ (or rather κ_s/κ_g) is a free parameter of the scalar gravity, treated as the scale of the local scale violation. The higher-derivative terms are to be suppressed by the powers of κ_s (or, possibly, κ_g). For generality we still explicitly retain the potential V_s (whose quadratic part gives a mass to systolon), assuming it to be the leading correction to the otherwise global symmetric gravity Lagrangian. The (light) systolon presents a scalar compression mode in metric in addition to the transverse-tensor, four-volume preserving deformation mode presented by the (massless) graviton. The systo-gravity, due to constraint (3) and the emergent spontaneous breaking of the compression symmetry, is apt to describe DM in the galaxy dark halos. To this end κ_s/κ_g ought to be of order $v_\infty/c \sim 10^{-3}$, where v_∞ is the asymptotic rotation velocity in galaxies, i.e., κ_s should lie in the GUT range, about 10^{15} GeV. Thus merging systo-gravity with GUT is natural.

3   Grand/hyper unification

At face value, the theory presented so far concerns the pure gravity. To account for matter let us address ourselves to a generic (renormalizable) GUT taking its general invariant Lagrangian as the matter one:

$$L_m = L_m(g_{\mu\nu}, V_\mu, \Phi, F). \tag{8}$$

Here V_μ, Φ and F are the generic gauge, scalar and chiral fermion fields, respectively. In particular, (8) includes a scalar potential $V_m(\Phi)$, whose (absolute) minimum defines the vacuum expectation values Φ_0's of Φ's. The detailed expressions are of no principle importance for what follows. Now switch on the systolon-matter interactions violating general relativity, with the unsuppressed unimodular invariant effective Lagrangian (of the canonical dimension $d = 4$) as follows:

$$\begin{aligned} L_{sm} &= \frac{1}{2}(\Sigma \xi_\Phi \Phi^\dagger \Phi) g^{\mu\nu} \partial_\mu \varsigma \partial_\nu \varsigma \\ &+ g^{\mu\nu}(i\Sigma \xi'_\Phi \Phi^\dagger \partial_\mu \Phi + \text{h.c.}) \partial_\nu \varsigma \\ &+ g^{\mu\nu}(\Sigma \xi_F \bar{F} \gamma_\mu F) \partial_\nu \varsigma. \end{aligned} \tag{9}$$

Here ξ_Φ, ξ'_Φ and ξ_F are some dimensionless parameters presumably of order unity. The gauge fields, which enter through the square of the covariant gauge strength $V_{\mu\nu}$ of the canonical dimension $d = 2$, do not admit a similar term. From the point of view of GUT alone, (9) is a priori as good as (8). In the case of absence of any other dimensionfull parameter, but for κ_g, all the higher-dimension terms ought to be suppressed by powers of κ_g.

After the spontaneous symmetry breaking, $\Phi = \Phi_0 + \Delta\Phi$, with $\Delta\Phi$'s being the physical scalar fields, one gets

$$\begin{aligned} L_{sm} &= \frac{1}{2}\Big(1 + \frac{1}{\kappa_s^2} \Delta N(\Phi_0, \Delta\Phi)\Big) g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \\ &+ \frac{1}{\kappa_s} g^{\mu\nu} J_\mu(\Phi_0, \Delta\Phi, F) \partial_\nu \sigma, \end{aligned} \tag{10}$$

where J_μ and ΔN are, respectively, the ensuing matter current and the correction to the kinetic term normalization. At that, κ_s gets not independent:[3]

$$\kappa_s^2 = \Sigma \xi_\Phi \Phi_0^\dagger \Phi_0. \tag{11}$$

Now we can completely abandon the kinetic term for σ in (7) in favour of (10). In this approach the pure gravity is general invariant (as in GR or in its general invariant modifications). Merging systo-gravity with grand unification may be referred to as the *hyper unification*. Above the scale κ_s the systolon, as a free propagating particle, disappears. For the finite field σ, its residual interactions with light particles in (10) are naturally suppressed at least as $\mathcal{O}(1/\kappa_s)$. The emergent decoupling of the systolon DM from the ordinary matter may serve as an additional reason in favour of the proposed hyper unification.

4 Conclusion

The inclusion of the local scale violation into the theory of particle interactions via GUT's seems theoretically very attractive as relating the drastically different distances in nature. If relevant, the hyper unification incorporating grand unification and systo-gravity, with systolon DM, may stretch down from the Planck scale to the cosmological distances. The phenomenological verification of the theory in cosmology, both at the large distances, notably with respect to the systolon DM, and at the high temperatures, would be crucial.[4]

References

[1] Yu. F. Pirogov, Eur. Phys. J. C **72** (2012) 2017; arXiv:1111.1437 [gr-qc].

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[3]Particularly, κ_s should depend on the state of the matter vacuum, e.g., its temperature.

[4]Under the hyper unification, the two-component DM, with a particle admixture to the systolon DM, is feasible. One would though expect the particle DM to be relevant (if any) near the core of dark halos, while the coherent systolon field off the core.